Microscopic Study of Quantum Vortex-Glass Transition Field in Two-Dimensional Superconductors

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The position $B_{\rm vg}$ of a field-tuned superconductor-insulator quantum transition occuring in disordered thin films is examined within the mean field approximation and starting from a hamiltonian of BCS type (favoring the s-wave pairing) with a random potential, and effects of an electron-electron repulsion on the transition field $B_{\rm vg}$ are also considered. Our calculation shows that the (microscopic) disorder-induced reduction of $B_{\rm vg}$ suggested experimentally cannot be explained without taking account of the familiar interplay between the randomness and the electron-electron repulsion enhancing the *quantum* superconducting fluctuation in such systems.

KEYWORDS: superconductor-insulator transition, vortex-glass transition, superconducting fluctuation

§1. Introduction

There have been many reports[1,2,3] on resistive data suggestive of a strong quantum fluctuation effect in disordered thin films under magnetic fields perpendicular to the film plane. In such systems, the temperature dependence of the sheet resistance $R(T)$ in higher fields than a field B_c is typically insulating, while in lower fields it decreases on cooling, in most cases, in a manner suggesting the presence of a magnetic field-tuned superconducting transition at zero temperature ($T = 0$). It was found experimentally[2,3] that B_c decreases with decreasing the film thickness or increasing the impurity density, both of which enhance the high temperature sheet resistance R_r.

Fisher[4] has proposed by extending the theoretical argument for zero field case[5] to nonzero fields ($B \neq 0$) that, at a field $B_{\rm vg}$ implying such a quantum superconducting transition point to be identified with B_c, the resistance curve $R(T)$ at lower temperatures should be independent of T and take a universal constant value. However, extending directly the description[5] in zero field to $B \neq 0$ case is theoretically invalid because a superconducting phase in the presence of the field-induced vortices is not the Meissner phase but a kind of a vortex glass (VG) phase[6,7] induced by a pinning disorder and with *no* Meissner effect. On the other hand, one of the present authors has pointed out[8] that, in the field range where the pinning disorder is negligible, the insulating behavior of resistance in $T \neq 0$ can intrinsically arise due to the quantum superconducting

fluctuation, and that, if the normal contribution σ_n to the conductivity is negligible, a flat $R(T)$ curve with approximately the value of quantum resistance $R_Q = \pi\hbar/2e^2 = 6.5(\mathrm{k}\Omega)$ is expected along the quantum melting (crossover) line at low temperatures which, as far as the ordinary dirty limit is valid, is insensitive to T. Also, it has been recently[7)] pointed out in terms of a phenomenological Ginzburg-Landau (GL) action with pinning disorder terms that, irrespective of the details of dynamics of the superconducting fluctuation, the disorder-induced contribution to the $T = 0$ conductance at $B_{\rm vg}$ should be, in contrast to the original argument,[4)] a *nonuniversal* constant dependent on the strengths of pinning and fluctuation.

However, in the present problem including a static disorder on the *electronic* level, a simple analysis starting from a GL action is insufficient, and more or less, one needs to return to a microscopic electronic hamiltonian. In a previous work,[9)] we have found, by deriving a quantum GL action, a significant enhancement of quantum fluctuation in nonzero fields due to an interplay between the microscopic disorder and a repulsive interaction between electrons. However, the familiar derivation of a GL action done there has neglected spatial variations of the coefficient of each term in the resulting GL model. A model of such spatial variations used widely in a GL theory is a random potential *for the pair-field* implying[7,10)] a randomness of T_{c0}, where T_{c0} is the mean field transition temperature in zero field and in clean limit. To clarify whether the idea[7,8)] based on the quantum superconducting fluctuation is applicable to real systems, we have to take account of such a GL random potential term which induces, in a finite field, a pinning disorder[11)] for the vortices. A crucial point is that, in the present issue, such a GL random potential arises from the *microscopic* disorder $u(\mathbf{r})$ which *simultaneously* enhances the (quantum) superconducting fluctuation. Note that, for a decrease and vanishing of resistivity in $B \neq 0$, the superconducting fluctuation effect and the vortex pinning are competitive with each other. Due to this competition originating from the same microscopic disorder, the disorder-induced decrease[1,2)] of $B_{\rm vg}$ may not be necessarily expected theoretically. The purpose of the present study is to give an answer to this question by starting from a microscopic model.

To examine the quantum superconducting fluctuation[8)] and the vortex glass fluctuation,[7)] arising from the pinning disorder, on the same footing, one needs an appropriate quantum GL action with spatially-dependent coefficients. Within the approximation taking account only of the pair-field (superconducting order parameter) Ψ belonging to the lowest Landau level (LLL), the quantum GL action is phenomenologically expected to take the following form

$$S_{\rm ran} = \int d^2r \left[\beta \sum_\omega (\mu(0) + \gamma|\omega|) \, |\Psi_\omega(\mathbf{r})|^2 + \int du \left(\delta\mu(\mathbf{r}) |\Psi(\mathbf{r}, u)|^2 + \frac{U_4}{2} |\Psi(\mathbf{r}, u)|^4 \right) \right], \qquad (1.1)$$

where u is an imaginary time, ω a Matsubara frequency, $\delta\mu(\mathbf{r})$ a static random potential implying a spatial variation of T_{c0}, $\Psi(\mathbf{r}, u)$ $(= N^{1/2}(0) < \psi_\uparrow(\mathbf{r}, u)\psi_\downarrow(\mathbf{r}, u) >) = \sum_\omega \Psi_\omega(\mathbf{r}) \exp(-{\rm i}\omega u)$, and γ measures[9)] the dissipation strength and was denoted as γ_1 in ref.9. Since we focused on the LLL

modes, eigenvalues of gradient terms were already absorbed into $\mu(0)$. For simplicity, a randomness of the quadratic term was taken in eq.(1.1) in a local form, i.e., up to the lowest order in the gradient. In §2, an action corresponding to a replicated version of the action (1.1) will be microscopically derived by neglecting the electron repulsion and focusing mainly on the low T and high B region. In §3, our formulation for determining the *Gaussian* vortex-glass transition field $B_{\rm vg,0}$ at $T=0$ will be explained and is applied both to the ordinary dirty limit with no repulsive interaction and to the case with an electron-repulsion. We find that, in contrast to the experimental observation,[1,2,3] $B_{\rm vg,0}$ in the ordinary dirty limit increases with increasing the (microscopic) disorder, mainly reflecting an enhancement of the corresponding $H_{c2}(0)$ ($= H_{c2}^d(0) \propto T_{c0}/\tau$). Further, the analysis is extended to the case with a repulsive interaction, and we find that the quantum LLL fluctuation and a decrease of $H_{c2}(0)$ due to an inclusion of an electron-repulsion are origins of a reduction of $B_{\rm vg}$ consistent with experimental observation. An attempt of computation on low T behaviors of γ is given in §4 to demonstrate the presence of a region in which the GL coefficients are insensitive to T. Our results are summarized in §5 together with a comment.

§2. Model and Derivation of Pinning Vertex

As a microscopic basis of our analysis, we first start from the s-wave BCS Hamiltonian with random potential in two dimensional (2D) case

$$H = \int_{\mathbf{r}} \left[\sum_\sigma \psi^\dagger_\sigma(\mathbf{r}) \left(\frac{\hbar^2}{2m} (-{\rm i}\nabla - \frac{\pi}{\phi_0}\mathbf{A}(\mathbf{r}))^2 + u(\mathbf{r}) \right) \psi_\sigma(\mathbf{r}) + g_{\rm BCS}\, \psi^\dagger_\uparrow(\mathbf{r})\psi^\dagger_\downarrow(\mathbf{r})\psi_\downarrow(\mathbf{r})\psi_\uparrow(\mathbf{r}) \right]. \qquad (2.1)$$

For the moment, we will not take account of electron-electron repulsions and postpone its inclusion to §3. In eq.(2.1), $g_{\rm BCS} < 0$, ϕ_0 is the flux quantum for Cooper pairs, ψ_σ is an annihilation operator of electron with spin σ ($=\uparrow, \downarrow$), and the applied field $\mathbf{B} = {\rm curl}\mathbf{A}$ is perpendicular to the 2 D plane. The random potential $u(\mathbf{r})$ has zero mean and the Gaussian ensemble $\overline{u(\mathbf{r})u(\mathbf{r}')} = (2\pi N(0)\tau)^{-1}\delta^{(2)}(\mathbf{r}-\mathbf{r}')$, where τ is the elastic relaxation time, $N(0)$ the density of states at Fermi surface, and the overbar denotes the microscopic random average. Following the study of zero field transition point in the literature,[12] the dependence of physical quantities on the film thickness is assumed to arise entirely through the repulsive interaction effect. In most part of this paper, our analysis is based on the ordinary dirty limit in 2D (with no interaction effect) in which the disorder strength is measured by $(k_F l)^{-1} = \tau/(2\pi N(0) l^2)$.

In our microscopic analysis which follows, the so-called quasi-classical approximation will be used. In this treatment, a gauge-invariant gradient $\mathbf{Q} = -{\rm i}\nabla + 2\pi\mathbf{A}/\phi_0$ operating on a Ψ is regarded as a wavevector (c-number) in the electronic process. Further, before focusing on the LLL modes of Ψ, the coefficients of the quadratic and quartic terms in the action are found to be functions of $\mathbf{Q}^2$, while $\mathbf{Q}^2$ is transformed, after operating on a LLL Ψ mode, into $r_B^{-2} \equiv 2\pi B/\phi_0$ where B is the uniform flux density. Hence, after this transformation $\mathbf{Q}^2 \to r_B^{-2}$, the coefficient of each term in the

action, except a term leading to the vortex pinning, may be written as a constant. In addtion, we note regarding the validity of the LLL approximation that, near $T = 0$, the field range over which the LLL approximation for the pair field is valid is quite broad even below $H_{c2}(0)$ according to ref.8. Under these conditions and within the ordinary dirty limit neglecting the localization effect and the electron-electron repulsion, the coefficients $\mu(0)$, γ, and U_4 in the resulting eq.(1.1) are well-known and given by[9,13]

$$U_4^{(0)} = \frac{8\pi\tau^3}{\beta N(0)} \sum_{\epsilon>0} (\,\Gamma(2\epsilon; B)\,)^3, \tag{2.2}$$

$$\mu^{(0)}(0) = (N(0)|g_{\rm BCS}|)^{-1} - 4\pi\tau\beta^{-1} \sum_{\epsilon>0} \Gamma(2\epsilon; B), \tag{2.3}$$

$$\gamma^{(0)} = 4\pi\beta^{-1}\tau^2 \sum_{\epsilon>0} (\,\Gamma(2\epsilon; B)\,)^2, \tag{2.4}$$

where $\Gamma(2\epsilon; B) = (2|\epsilon| + 2\pi BD/\phi_0)^{-1}\tau^{-1}$ implies $\Gamma_{\mathbf{Q}}(2\epsilon)$ with $Q^2 = 2\pi B/\phi_0$, $\Gamma_{\mathbf{q}}(\epsilon+\epsilon') = \tau^{-1}(|\epsilon+\epsilon'| + Dq^2)^{-1}$ is the diffusion propagator with momentum $\mathbf{q}$, ϵ and ϵ' are Matsubara frequencies for fermions, and the diffusion constant D is given in terms of the mean free path $l = k_F\tau/m$ by $D = l^2/(2\tau)$. Through this paper, the (impurity-averaged) electron propagator is, as usual, given by $G_{\mathbf{k}}(\epsilon) = ({\rm i}\epsilon - \xi_k + ({\rm i}/2\tau){\rm sgn}(\epsilon))^{-1}$.

Next, to explain how the $\delta\mu(\mathbf{r})$ term, neglected in previous studies,[9] appears in the present formulation, let us first imagine the phenomenological action (1.1) to be rewritten in a replicated form. Following the standard treatment for an action of GL type, the replicated form[10] of eq.(1.1) will be given by

$$S^n_{\rm ran} = \sum_{\alpha=1}^{n} \Biggl[\int_{\mathbf{r}} \beta \sum_{\omega} (\mu(0) + \gamma|\omega|)\, |\Psi^{(\alpha)}_{\omega}(\mathbf{r})|^2 + \frac{U_4}{2} \int du \int_{\mathbf{k}} |\rho^{(\alpha)}_{\mathbf{k}}(u)|^2$$

$$- \sum_{\alpha'=1}^{n} \int du_1 \int du_2 \int_{\mathbf{k}} \frac{U_p(\mathbf{k})}{2}\, \rho^{(\alpha)}_{\mathbf{k}}(u_1) \rho^{(\alpha')}_{-\mathbf{k}}(u_2) \Biggr], \tag{2.5}$$

where α and α' denote the replica indices, u_1 and u_2 are imaginary times, and the bare ($\mathbf{k}$-dependent) pinning vertex function $U_p(\mathbf{k})$ and $\rho^{(\alpha)}_{\mathbf{k}}(u)$ are Fourier-transformations, respectively, of $\overline{\delta\mu(\mathbf{r})\,\delta\mu(\mathbf{r'})}$ and of $|\Psi^{(\alpha)}(\mathbf{r}, u)|^2$. As seen below, the $\mathbf{k}$-dependence of the bare pinning vertex $U_p(\mathbf{k})$ is characteristic of the high B and low T region.

A microscopic derivation of the last term of eq.(2.5) will be explained here. Within the quasiclassical approximation,[13] the third (replica off-diagonal) term of eq.(2.5) generally takes the following form

$$\Delta S^n_{\rm ran} = -\frac{\beta^2}{2} \sum_{\alpha=1}^{n} \sum_{\alpha'=1}^{n} \int d^2r \sum_{\omega} \sum_{\omega'} \mathcal{F}([\mathbf{Q}_s];\, \omega, \omega')$$

$$\times (\Psi^{(\alpha)}_{\omega}(\mathbf{r}_1))^*\, \Psi^{(\alpha)}_{\omega}(\mathbf{r}_2)\, (\Psi^{(\alpha')}_{\omega'}(\mathbf{r}_3))^*\, \Psi^{(\alpha')}_{\omega'}(\mathbf{r}_4) \Big|_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4 \to \mathbf{r}}, \tag{2.6}$$

where $\mathbf{Q}_s = -\mathrm{i}\nabla_s + (2\pi/\phi_0)\mathbf{A}(\mathbf{r}_s)$, $\mathcal{F}([\mathbf{Q}_s])$ is a bare vertex of the quartic term induced by the random-average, and $[\mathbf{Q}_s]$ implies dependences on $\mathbf{Q}_1^*$, $\mathbf{Q}_2$, $\mathbf{Q}_3^*$, and $\mathbf{Q}_4$. Eq.(2.6) includes terms off-diagonal in the replica indices as a reflection of the fact that no impurity lines carry finite frequencies. Due to the use of the quasiclassical approximation, the spatial nonlocality, arising from the nonzero magnetic field, in $\mathcal{F}(\cdots)$ appears only in the form of the gauge-invariant gradients $\mathbf{Q}_s$ operating the pair-fields $\Psi(\mathbf{r}_s)$. Hereafter, frequency dependences of $\mathcal{F}$ will be neglected.

Below, let us see how ΔS^n_{ran} will lead to the form of the last term of eq.(2.5) in a consistent approximation with the derivations of $U_4^{(0)}$, $\mu^{(0)}(0)$, and $\gamma^{(0)}$. Namely, as in the derivation of $U_4^{(0)}$, the diagrams expressing $\mathcal{F}$ are selected under the condition, equivalent to the neglect of localization effect of noninteracting electrons, that the impurity lines, each of which carries the factor $(2\pi N(0)\tau)^{-1}$, do not cross to each other in the diagrams. The diagrams thus obtained expressing $\mathcal{F}$ are the same as those in ref.14, where the diagrams were examined from the viewpoint of mesoscopic fluctuation, and their examples belonging to the same family are given in Fig.1. Effects of an electron-repulsion on ΔS^n_{ran} will not be examined here. We simply note that, just like those in U_4, each term perturbative in a short-ranged repulsive interaction in the function $\mathcal{F}([\mathbf{Q}_s])$ is convergent in $T \to 0$ limit. The first three diagrams of those in Fig.1 are given in Fig.2, on which we will focus here to clarify key features common to all diagrams in Fig.1. In the figures, the single solid curves denote $G_k(\epsilon)$'s, four double solid lines denote $\Psi(\mathbf{r}_i)$ or $\Psi^*(\mathbf{r}_i)$, and $\mathbf{Q}_i$ ($\mathbf{Q}_i^*$) operates on $\Psi(\mathbf{r}_i)$ ($\Psi^*(\mathbf{r}_i)$). The sum of diagrams in Fig.2 contributes to $\mathcal{F}([\mathbf{Q}_s])$ in the manner

$$\mathcal{F}^{(2)}([\mathbf{Q}_s]) = (N(0)\beta)^{-2}\sum_{\epsilon>0}\sum_{\epsilon'>0}\Gamma_{\mathbf{Q}_1^*}(2\epsilon)\Gamma_{\mathbf{Q}_2}(2\epsilon)\Gamma_{\mathbf{Q}_3^*}(2\epsilon')\Gamma_{\mathbf{Q}_4}(2\epsilon')\,(2\pi N(0)\tau)^{-2}\int_{\mathbf{q}}\Gamma_{\mathbf{q}+\mathbf{Q}_3^*-\mathbf{Q}_2}(\epsilon+\epsilon')$$

$$\times\Gamma_{\mathbf{q}}(\epsilon+\epsilon')\int_{\mathbf{k}} G_{\mathbf{k}+\mathbf{Q}_2}(\epsilon)G_{\mathbf{k}}(-\epsilon)G_{\mathbf{k}+\mathbf{Q}_3^*+\mathbf{q}}(-\epsilon')G_{\mathbf{k}+\mathbf{q}}(\epsilon')\,(I^{(2a)}+I^{(2b)}+I^{(2c)}), \tag{2.7}$$

where

$$I^{(2a)} = \int_{\mathbf{k}} G_{\mathbf{k}+\mathbf{Q}_1^*}(\epsilon)G_{\mathbf{k}}(-\epsilon)G_{\mathbf{k}+\mathbf{q}+\mathbf{Q}_4}(-\epsilon')G_{\mathbf{k}+\mathbf{q}}(\epsilon'),$$

$$I^{(2b)} = -(2\pi\tau N(0))^{-1}\int_{\mathbf{k}_1} G_{\mathbf{k}_1+\mathbf{Q}_1^*}(\epsilon)G_{\mathbf{k}_1}(-\epsilon)G_{\mathbf{k}_1+\mathbf{q}}(\epsilon')\int_{\mathbf{k}_2} G_{\mathbf{k}_2+\mathbf{Q}_1^*}(\epsilon)G_{\mathbf{k}_2+\mathbf{q}+\mathbf{Q}_4}(-\epsilon')G_{\mathbf{k}_2+\mathbf{q}}(\epsilon'),$$

and

$$I^{(2c)} = -(2\pi\tau N(0))^{-1}\int_{\mathbf{k}_1} G_{\mathbf{k}_1+\mathbf{Q}_1^*}(\epsilon)G_{\mathbf{k}_1}(-\epsilon)G_{\mathbf{k}_1+\mathbf{q}+\mathbf{Q}_4}(-\epsilon')\int_{\mathbf{k}_2} G_{\mathbf{k}_2}(-\epsilon)G_{\mathbf{k}_2+\mathbf{q}+\mathbf{Q}_4}(-\epsilon')G_{\mathbf{k}_2+\mathbf{q}}(\epsilon'), \tag{2.8}$$

and $I^{(2a)}$ is the contribution of Fig.2(a) and so forth. However, a cancellation occurs between the three diagrams in the manner that the sum $I^{(2a)} + I^{(2b)} + I^{(2)}$ reduces to $2\pi N(0)\tau^4(2\epsilon + 2\epsilon' + D(Q_1^*)^2 + DQ_4^2 + 2D(q^2 + \mathbf{q}\cdot(\mathbf{Q}_4 - \mathbf{Q}_1^*)))$. It is easily understood that one more cancellation similar to this arises when all diagrams of Fig.1 are summed up, and the contribution of Fig.1 to $\mathcal{F}$ becomes

$$\mathcal{F}^{(1)}(\mathbf{Q},\Delta\mathbf{Q}) = \left(\frac{2\tau^2}{\beta N(0)}\right)^2\sum_{\epsilon>0}\sum_{\epsilon'>0}(\,\Gamma_{\mathbf{Q}}(2\epsilon)\Gamma_{\mathbf{Q}}(2\epsilon')\,)^2\int_{\mathbf{q}}\Gamma_{\mathbf{q}+\Delta\mathbf{Q}}(\epsilon+\epsilon')\,\Gamma_{\mathbf{q}}(\epsilon+\epsilon')$$

$$\times(\epsilon+\epsilon'+DQ^2+Dq^2+D\mathbf{q}\cdot\Delta\mathbf{Q})^2\tau^2, \tag{2.9}$$

where $\Delta\mathbf{Q} = \mathbf{Q}_4 - \mathbf{Q}_1^* = \mathbf{Q}_3^* - \mathbf{Q}_2$, and all $\mathbf{Q}_s^2$ and $(\mathbf{Q}_s^*)^2$ were expressed as $\mathbf{Q}^2$, because the external pair-fields are assumed to be in LLL. Further, using the fact that, after operating a LLL eigenfunction, $\mathbf{Q}^2$ changes into $r_B^{-2} = 2\pi B/\phi_0$, eq.(2.9) implies that $\mathcal{F}^{(1)}$ takes the form $\mathcal{F}^{(1)}(\Delta\mathbf{Q}) = U_p f^{(1)}(\,t;\, r_B\Delta\mathbf{Q}\,)$, where

$$U_p = \left(\frac{4}{\pi}\frac{r_B\,\tau}{N(0)l^2}\right)^2, \tag{2.10}$$

and $t = 8\pi\tau/(\beta l^2 r_B^{-2})$. Although other families of diagrams are also found to have similar structures to $\mathcal{F}^{(1)}$, the full expression $\mathcal{F}$ obtained after summing them up has a highly complicated $\Delta\mathbf{Q}$-dependence. Here we merely mention that, if formally setting $\Delta\mathbf{Q} = 0$, the full $\mathcal{F}$ becomes the simplified form:

$$\mathcal{F}(\Delta\mathbf{Q}=0) \simeq 12\left(\frac{\tau^2}{N(0)\beta}\right)^2 \sum_{\epsilon>0}\sum_{\epsilon'>0}\Gamma_{\mathbf{Q}}(2\epsilon)\Gamma_{\mathbf{Q}}(2\epsilon')\int_{\mathbf{q}}(\Gamma_{\mathbf{q}}(\epsilon+\epsilon'))^2, \tag{2.11}$$

which is estimated as $0.14U_p$ in low t limit. Since, as is seen below, it is difficult to write down a concrete expression of $U_p(\mathbf{k})$ resulting from the full $\mathcal{F}(\Delta\mathbf{Q})$, we will merely explain below some properties of the full $U_p(\mathbf{k})$.

At this stage, let us rewrite the terms of eq.(2.5) entirely in terms of the LLL fluctuation field $\varphi_0^{(\alpha)}(p,\omega)$ so that $\rho_{\mathbf{k}}^{(\alpha)}(u) = \sum_\omega \rho^{(\alpha)}(\mathbf{k},\omega)e^{-\mathrm{i}\omega u}$ is expressed in a Landau gauge, by

$$\rho^{(\alpha)}(\mathbf{k},\Omega) = \sum_p \exp(-\frac{\mathbf{k}^2 r_B^2}{4} + \mathrm{i}pk_x r_B^2)\sum_\omega (\varphi_0^{(\alpha)}(p+k_y/2,\omega+\Omega))^*\,\varphi_0^{(\alpha)}(p-k_y/2,\omega), \tag{2.12}$$

where $\varphi_0^{(\alpha)}(p,\omega)$ is defined as $\Psi_\omega(\mathbf{r}) = \sum_p \varphi_0(p,\omega)u_{0,p}(\mathbf{r})$ in terms of a LLL eigen function $u_{0,p}$. If the $\Delta\mathbf{Q}$-dependences in $\mathcal{F}$ are neglected, the resulting pining vertex is local (i.e., the function $U_p(\mathbf{k})$ is $\mathbf{k}$-independent). This approximation is valid in high T and low B region defined by $t \gg 1$, i.e., $T \gg 0.15T_{c0}B/H_{c2}^d(0)$. In contrast, at low T and high B of our interest, any microscopic length on the pair-fields is scaled by r_B, and thus, the $\Delta\mathbf{Q}$-dependences are no longer negligible. Actually, we have the relations in a Landau gauge

$$\int_{\mathbf{r}}\Delta\mathbf{Q}(\Psi^{(\alpha)}(\mathbf{r}_1))^*\Psi^{(\alpha)}(\mathbf{r}_2)(\Psi^{(\alpha')}(\mathbf{r}_3))^*\Psi^{(\alpha')}(\mathbf{r}_4)|_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4\to\mathbf{r}}$$

$$= \mathrm{i}\sum_{p,p'}\int_{\mathbf{k}}(\hat{z}\times\mathbf{k})\,v_{\mathbf{k}}e^{\mathrm{i}(p-p')k_x r_B^2}(\varphi_0^{(\alpha)}(p-k_y/2))^*\varphi_0^{(\alpha)}(p+k_y/2)(\varphi_0^{(\alpha')}(p'+k_y/2))^*\varphi_0^{(\alpha')}(p'-k_y/2),$$

$$\int_{\mathbf{r}}(\Delta\mathbf{Q})^2(\Psi^{(\alpha)}(\mathbf{r}_1))^*\Psi^{(\alpha)}(\mathbf{r}_2)(\Psi^{(\alpha')}(\mathbf{r}_3))^*\Psi^{(\alpha')}(\mathbf{r}_4)|_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4\to\mathbf{r}}$$

$$= 2\sum_{p,p'}\int_{\mathbf{k}}\left(r_B^{-2}-\frac{\mathbf{k}^2}{2}\right)v_{\mathbf{k}}e^{\mathrm{i}(p-p')k_x r_B^2}(\varphi_0^{(\alpha)}(p-k_y/2))^*\varphi_0^{(\alpha)}(p+k_y/2)(\varphi_0^{(\alpha')}(p'+k_y/2))^*\varphi_0^{(\alpha')}(p'-k_y/2) \tag{2.13}$$

and so on, where $v_{\mathbf{q}} = \exp(-\mathbf{q}^2 r_B^2/2)$. Namely, any $r_B \Delta\mathbf{Q}$-dependence changes, after the Landau level representation, into a $r_B\mathbf{k}$-dependence or a constant. Hence, $U_p(\mathbf{k})$ in eq.(2.5) generally has the form

$$U_p(\mathbf{k}) = U_p\, f_{00}(t;\, \mathbf{k}r_B). \tag{2.14}$$

If the Fermi surface is isotropic, f_{00} is a function of $k^2 r_B^2$. Below, for brevity, we will focus on this isotropic case. At low temperatures $t \ll 1$, f_{00} is not sensitive to the material parameters included in the definition of t but depends merely on $k^2 r_B^2$. Unfortunately, it is difficult to find an exact form of such k-dependences in terms of eq.(2.13) which implies that a $\Delta\mathbf{Q}$-expansion does not reduce to a systematic $\mathbf{k}$-expansion. Nevertheless, we have tried to estimate the k-dependence of f_{00} at $T = 0$ occuring from the low frequency limit of frequency-integrals, because the corresponding dependence occuring from higher frequencies is clearly regular, and a large k-contribution is cut off by the $v_{\mathbf{k}}$-factor appearing everywhere in the LLL diagrammatics. Using the relations (2.13), a nonanalytic k-dependence arising from the lowest frequencies is found to be less singular than $kr_B \ln(kr_B)^{-1}$ which vanishes in $k \to 0$. We note that, since $\mathbf{k}$ is always accompanied by r_B, the pinning strength occuring after a $\mathbf{k}$-integral in a physical quantity is measured by $U_p/r_B^2 \simeq (E_{\rm F}\tau)^{-2}$ in the dirty limit in $t < 1$. Since the quantum fluctuation strength $U_4/(r_B^2\gamma)$ becomes, as seen below, of the order $(E_{\rm F}\tau)^{-1}$ in the dirty limit, this implies that the relative (effective) pinning strength $U_p\gamma/U_4$ is $\mathrm{O}((E_{\rm F}\tau)^{-1})$. It is verified in terms of eqs.(2.2) and (2.11) in $t \gg 1$ limit that this is of the same order as the thermal counterpart, denoted as $\Delta_{\rm eff}$ in ref.15, $\beta U_p(t \gg 1; \mathbf{k} = 0)/U_4$ in the dirty limit. Therefore, in the ordinary dirty limit, the vortex pinning strength relatively *increases* at any T with increasing disorder with strength $(E_{\rm F}\tau)^{-1}$.

§3. Quantum VG Transition Point

In this section, we examine the $T = 0$ VG transition point $B_{\rm vg,0}$ in the Gaussian (i.e., mean field) approximation using the LLL-GL action in the dirty limit derived in §2, and, based on this result, effects of the electron-electron repulsion on $B_{\rm vg,0}$ are qualitatively studied. The VG transition field is defined[4,6,7] by examining where the uniform VG susceptibility $\chi_{\rm vg}$ (defined below) at zero frequency tends to diverge. For a moment (until reaching eq.(3.16)), the GL action is assumed to be well-defined in low T limit.

The key quantity $\chi_{\rm vg}$ in this section is proportional to $\int_{\mathbf{r}_1}\int_{\mathbf{r}_2} \overline{| < \Psi_{\omega=0}(\mathbf{r}_1)\Psi^*_{\omega=0}(\mathbf{r}_2) > |^2}$ and, consistently with eqs.(2.5) and (2.11), is given by

$$\chi_{\rm vg} = \sum_{p-p'} \overline{|\mathcal{G}(p, p',\, 0)|^2}, \tag{3.1}$$

where $\mathcal{G}(p, p',\, \omega) =< \varphi_0(p, \omega)\, \varphi_0^*(p', \omega) >$ is the LLL fluctuation propagator defined prior to the random averaging. On the other hand, the renormalized 2D LLL superconducting fluctuation,

defined through the random-averaged (replica-diagonal) fluctuation propagator

$$\mathcal{G}(\omega) = \overline{\mathcal{G}(p, p', \omega)} = (\, \mu(0) + \gamma|\omega| + \Sigma(\omega)\,)^{-1} \tag{3.2}$$

with self energy $\Sigma(\omega)$, is nondivergent (noncritical) even at $T = 0$ below the mean field $H_{c2}(0)$, because the dimensionality of the LLL fluctuation in 2D is two even at $T = 0$. Hence, it erases the *mean field* superconducting transition on the $H_{c2}(T)$-line to create a first order vortex-solidification transition below it.[15,16] In 3D, on the other hand, the dimensionality of dissipative LLL fluctuation at $T = 0$ becomes three, and hence, a "ctitical field" $H^*_{c2}(0)$ can be defined at $T = 0$ *below* the solidification field $B_m(0)$ (see §5 in ref.8).

If the (Gaussian) VG transition occurs in a *higher* field than the disorder-free true transition (i.e., the solidification transition) point, a familiar resummation approximation of diagrams, such as RPA, is applicable[15] in adressing a glass transition point. Following previous works, we will invoke a systematic loop (or $1/M$) expansion and focus on its lowest order ($M = \infty$) terms by, as a mathematical tool, assuming Ψ to have M-flavors. It is because this approach applied to the 3 D *thermal* vortex state under a correlated (line-like) disorder parallel to the field has led[7] to a transition line qualitatively consistent with the superconducting transition in high Tc superconductors with parallel columnar defects. Note that, since the disorder with strength U_p is, in eq.(3.1) at $T = 0$, persistently correlated along the "time" direction, the present quantum GL model with disorder is formally similar,[7] close to $T = 0$, to the bulk 3 D GL model at high temperatures with line-like defects parallel to the field.

Hereafter, we write $\Sigma(\omega) = \Sigma_{\rm vg}(\omega) + \Sigma_{\rm F}$, where the first and second terms are, respectively, given by Fig.3 (a) and Fig.3 (b). The expression of $\chi_{\rm vg}$ consistent with them has the form of a ladder of pinning lines with vertex correction, as described in Fig.3 (c) and (d). In expressing the diagrams explicitly, the bare pinning strength $U_p/(2\pi r_B^2)$ can be regarded as being replaced by

$$\Delta_0^{(R)} = U_p \int_{\mathbf{k}} v_{\mathbf{k}}\, f_{00}(k^2 r_B^2) \left(1 + \sigma_{\rm vg}\, v_{\mathbf{k}} \right)^{-2} = \frac{U_p}{2\pi r_B^2} C(\sigma_{\rm vg}), \tag{3.3}$$

where the factor $C(\sigma_{\rm vg}) = \int_0^\infty dkk\, f_{00}(k^2) e^{-k^2/2} (1 + \sigma_{\rm vg} e^{-k^2/2})^{-2}$ with

$$\sigma_{\rm vg} = \frac{U_4}{2\pi r_B^2 \beta} \sum_{\omega''} (\, \mathcal{G}(\omega'')\,)^2 \tag{3.4}$$

implies a renormalization (vertex correction) due to the LLL fluctuation of the pinning strength. The detailed form of $C(\sigma_{\rm vg})$ depends on the functional form of f_{00} and thus, as mentioned in §2, is not known concretely. We just expect $f_{00}(\mathbf{k})$ to have an algebraic form in k^2. It implies that, when $\sigma_{\rm vg} \gg 1$, $\Delta_0^{(R)}$ will roughly behave like $\sim \sigma_{\rm vg}^{-1} U_p / r_B^2$. The two self energy terms are given by

$$\Sigma_{\rm vg}(\omega) = -\Delta_0^{(R)}\, \mathcal{G}(\omega), \tag{3.5}$$

$$\Sigma_{\rm F} = \frac{U_4}{2\beta} \sum_\omega \int_{\mathbf{k}} v_{\mathbf{k}} \mathcal{G}(\omega) = \frac{U_4}{4\pi r_B^2 \beta} \sum_\omega \mathcal{G}(\omega), \tag{3.6}$$

and thus, eq.(3.2) yields

$$(\,\mathcal{G}(\omega)\,)^{-1} = \mu(0) + \gamma|\omega| - \Delta_0^{(R)} \mathcal{G}(\omega) + \Sigma_{\rm F}. \tag{3.7}$$

The Gaussian VG susceptibility $\chi_{\rm vg}$, consistent with the above $\Sigma_{\rm vg}(\omega)$, is given as a series of ladder diagrams. Since the propagator $\mathcal{G}(\omega)$, due to the LLL degeneracy, depends only on a Matsubara frequency ω, the sum of ladder diagrams is a trivial geometrical series, and hence, $\chi_{\rm vg}$ simply becomes

$$\chi_{\rm vg} = 2\pi r_B^2 \Delta_0^{(R)} \left(1 - \Delta_0^{(R)} \mathcal{G}^2(0) \right)^{-1} \tag{3.8}$$

so that the "Gaussian" VG transition point is given by the equation

$$\Delta_0^{(R)} \mathcal{G}^2(0) = \frac{U_p\, C(\sigma_{\rm vg})}{2\pi r_B^2} \mathcal{G}^2(0) = 1. \tag{3.9}$$

Eq.(3.7) is quadratic in the renormalized propagator $\mathcal{G}(\omega)$ and easily analyzed. Further, at $T = 0$ where the frequency summation is replaced by an integral, we can accomplish the present analytic calculation. Using eq.(3.9), the propagator $\mathcal{G}(\omega)$ resulting from eq.(3.7) is expressed at the VG transition field $B_{\rm vg,0}$ by

$$\mathcal{G}^{-1}(\omega) = \mathcal{G}^{-1}(0) + \frac{\gamma|\omega|}{2} + \sqrt{\gamma|\omega| \left(\mathcal{G}^{-1}(0) + \frac{\gamma|\omega|}{4} \right)} \tag{3.10}$$

with

$$2\,\mathcal{G}^{-1}(0) = \ln\left(\frac{B_{\rm vg,0}}{H_{c2}^d(0)} \right) + \Sigma_F, \tag{3.11}$$

where eq.(3.9) was used. The $\sigma_{\rm vg}$-expression at $B_{\rm vg,0}$ is expressed as

$$\sigma_{\rm vg} = \frac{U_4}{4\pi^2 r_B^2} \int_0^\infty d\omega \mathcal{G}^2(\omega) = \frac{U_4}{6\pi^2 \gamma r_B^2} \mathcal{G}(0), \tag{3.12}$$

and, by combining this with eq.(3.9), $\sigma_{\rm vg}$ is a function of the combination

$$\eta = \frac{U_4^2}{18\pi^3 \gamma^2 r_B^2 U_p} \tag{3.13}$$

and given as a solution of

$$\sigma_{\rm vg}^2\, C(\sigma_{\rm vg}) = \eta. \tag{3.14}$$

On the other hand, the integral of Σ_F requires a high-frequency cut off. By reasonably assuming a constant of order unity $\Lambda_c \sim \gamma\omega_M$, where ω_M is a frequency cut off, the frequency sum (integral) of eq.(3.6) results in

$$\Sigma_F = \frac{U_4}{4\pi^2 r_B^2 \gamma} \ln\left(\frac{6\pi^2 \gamma r_B^2 \sigma_{\rm vg} \Lambda_c}{U_4} \right). \tag{3.15}$$

Applying this to eq.(3.11), we obtain the relation

$$\ln\left(\frac{B_{\rm vg,0}}{H_{c2}^d(0)}\right) = \frac{U_4}{4\pi^2 r_B^2 \gamma}\left(\frac{4}{3\sigma_{\rm vg}} - \ln\left(\frac{6\pi^2\gamma r_B^2 \sigma_{\rm vg}\Lambda_c}{U_4}\right)\right). \tag{3.16}$$

Eqs.(3.13), (3.14), and (3.16) give the $T=0$ VG transition field $B_{\rm vg,0}$. In the bracket of r.h.s. of eq.(3.16), the first term is a measure of the vortex pinning strength, while the second term is a measure of quantum LLL fluctuation effect.

First, let us apply the above result to the ordinary dirty limit. According to eqs.(2.2) and (2.4), the coefficients in GL action take the following values in $T \to 0$ limit

$$\gamma \to \gamma^{(0)}(T=0) = \frac{\tau\phi_0}{\pi B\, l^2},$$
$$U_4 \to N^{-1}(0)\,(\gamma^{(0)}(T=0))^2, \tag{3.17}$$

together with eq.(2.10). Consequently, in this case, η is a universal number, 5×10^{-3}, and hence, $\sigma_{\rm vg}$ is also a constant, possibly of the order of 10^{-1}. Further, the strength $U_4/(4\pi^2 r_B^2\gamma)$ of quantum superconducting fluctuation becomes $(2\pi E_{\rm F}\tau)^{-1}$. Then, eq.(3.16) implies that, although $B_{\rm vg,0}$ will lie below $H_{c2}^d(0)$ for large enough $E_{\rm F}\tau$, it *increases* with increasing the disorder strength $1/(E_{\rm F}\tau)$. One can find that the primary origin of this $B_{\rm vg,0}$-increase is the disorder-induced enhancement of $H_{c2}^d(0) \propto \tau^{-1}$, while the cancellation between the two terms in the bracket of eq.(3.16) is subtle and may depend on the diagram-resummation method which should be refined in higher orders of $(E_{\rm F}\tau)^{-1}$. Here we will respect the present result in the simple but *consistent* $M^{-1}=0$ approximation and expect, more generally (but in the dirty limit), the fluctuation contribution to outweigh the pinning contribution and to make $B_{\rm vg,0}$ lower than $H_{c2}^d(0)$ at least up to $\mathrm{O}((E_{\rm F}\tau)^{-1})$ (see also the next paragraph). On the other hand, *without* the fluctuation (second) term in eq.(3.16), it would show a $T=0$ superconducting transition point increasing with increasing disorder and existing *above* $H_{c2}^d(0)$. This statement is essentially the same as the argument by Spivak and Zhou[14] of an unlimitedly large "H_{c2}" (see also the sentence prior to eq.(2.7)). As already mentioned, we expect the inclusion of the quantum superconducting fluctuation to push the superconducting transition point down to $B_{\rm vg,0}$ below $H_{c2}^d(0)$, even in the ordinary dirty limit.

The true transition point $B_{\rm vg}$ should be lowered further from the Gaussian one $B_{\rm vg,0}$ by going beyond the present Gaussian approximation and taking account of interactions between the VG fluctuations. Explaining this requires another apparatus and will be given in a separate paper.[17] We note here that this shift $1 - B_{\rm vg}/B_{\rm vg,0}$ is also of the order of $(E_{\rm F}\tau)^{-1}$. Nevertheless, this fact does not change the above statement in the ordinary dirty limit that $B_{\rm vg} < H_{c2}^d(0)$, while $B_{\rm vg}$ will increase with increasing $(E_{\rm F}\tau)^{-1}$. Below, we will not distinguish $B_{\rm vg}$ from $B_{\rm vg,0}$ in discussing effects of an electron-repulsion. This simplification does not affect conclusions which follow in this section.

Now, let us examine how an interplay between an electron-repulsion and disorder changes the above results in the ordinary dirty limit. According to the previous works[12)] (see also ref.19), the contributions of the dynamically-screened Coulomb interaction to the linearized GL equation for quasi 2D films can be included perturbatively by assuming in 2D case a short-ranged repulsive interaction with strength $\lambda_1 = R_r/(8\pi R_Q) = 3/(k_{\rm F}^2 \, d \, l)$, where d is the film thickness, and R_r is identified with the high temperature sheet resistance. As shown in ref.9, the corrections due to the short-ranged electron repulsion to the GL coefficients $\mu(0)$ and U_4 are convergent in low T limit at each order in λ_1, since the frequency dependences of Cooperons appearing as vertex corrections for the couplings to the pair-fields are cut off by the B-dependence. It implies that $\mu(0)$ and U_4 are determined by the high frequency side of Matsubara frequency summations and that their T dependences will, irrespective of λ_1-values, be lost roughly when $t < 1$, i.e., $T < T_{cr}^{\rm mf} \equiv 0.15 T_{c0} B/H_{c2}^d(0)$ (see §2). As mentioned above eq.(2.7), the situation is also similar in λ_1-dependences of $U_p f_{00}({\bf k} r_B)$. In contrast, each term of λ_1-perturbation series for the time scale γ is logarithmically divergent, and, at low enough T, γ has the systematic expansion parameter[9,18)] $\lambda_1 \ln(T/T_{cr}^{\rm mf})$. It is expected from this systematic perturbation series that an onset temperature below which γ begins to rapidly decrease on cooling will be given by

$$T_{\rm rep}(\lambda_1) \simeq T_{cr}^{\rm mf} \exp(-c_{\rm rep}/\lambda_1), \tag{3.18}$$

where $c_{\rm rep}$ is a positive constant (possibly) slightly less than unity. These facts imply that we have an intermediate temperature region below $T_{cr}^{\rm mf}$ but above $T_{\rm rep}(\lambda_1)$. Besides these *microscopic* temperature scales, we have the quantum-thermal crossover temperature $T_{cr} \equiv U_4/(2\pi r_B^2 \gamma^2)$ on the LLL fluctuation behavior (see §2 in ref.8). Then, if $T_{\rm rep} \ll T_{cr}$ $(< T_{cr}^{\rm mf})$, there is a low but intermediate temperature region above $T_{\rm rep}$ but below T_{cr} in which any microscopic T-dependence carried by the GL coefficients is negligible and the LLL fluctuation is of quantum nature, although the GL coefficients are different from those in the dirty limit due to the λ_1 corrections. In §4, a computational evidence on the presence of such an intermediate temperature range will be given. Then, an *apparent* VG transition field $B_{\rm vg}^*$ can be *defined* in this intermediate temperature region as a "$T = 0$" transition field and, according to eq.(3.16), is expressed by

$$B_{\rm vg}^* \simeq H_{c2}(0)(1 - d_{\rm g}^* (E_{\rm F}\tau)^{-1}), \tag{3.19}$$

up to O($(E_{\rm F}\tau)^{-1}$), where $H_{c2}(0)$ and the constant $d_{\rm g}^*$ have λ_1-corrections. Although we expect $d_{\rm g}^*$ to be positive, it is possibly smaller than unity, at least when $\lambda_1 = 0$, once recalling the above-mentioned cancellation between a fluctuation term and a pinning term. Although the λ_1-correction to $d_{\rm g}^*$ is due to those in U_4 and U_p, it is not easy to exactly obtain those corrections even up to O(λ_1). However, since the λ_1-correction in $d_{\rm g}^*$ is accompanied in eq.(3.19) by the factor $1/E_{\rm F}\tau$, the λ_1-dependence of $B_{\rm vg}^*$ can be seen as being dominated by that of $H_{c2}(0)$. It is now known that $H_{c2}(0)$ decreases[9,19)] with increasing R_r, and thus, the *apparent* critical field $B_{\rm vg}^*$, defined in

the intermediate temperature range, is expected to decrease with increasing R_r, as indicated in experiments.[1,2,3]

To find the R_r-dependence of a *true* (but, possibly, inaccessible) $B_{\rm vg}$ at $T = 0$, we need the λ_1-dependences of the GL coefficients in low T limit $T \ll T_{\rm rep}$. Since, at the present stage, we have no computation evidence enough to argue that γ will remain positive in low T limit, we will merely assume here $\gamma(T \to 0)$ to approach a small but positive value $\gamma_{\rm min}$. Then, according to eq.(3.16), $B_{\rm vg}$ is given by eq.(3.19) with $d_{\rm g}^*$ there replaced by $d_{\rm g}$, which will take the form

$$d_{\rm g} \simeq \frac{\gamma^{(0)}}{2\pi\, \gamma_{\rm min}} \ln\left(\frac{\gamma^{(0)}}{\gamma_{\rm min}}\right) \gg d_{\rm g}^*, \tag{3.20}$$

where other $\gamma_{\rm min}$-independent terms were neglected by assuming $\gamma_{\rm min} \ll \gamma^{(0)}$. Since it will be clear that $\gamma_{\rm min}$ is insensitive to λ_1 or decreases with increasing λ_1, $B_{\rm vg}$ will decrease, more drastically than $B_{\rm vg}^*$, with increasing R_r. However, we believe through the results in §4 and experimental informations that the field B_c at which a flat (T-independent) resistance curve is seen should correspond to $B_{\rm vg}^*$ and that $B_{\rm vg}$ is not measurable because $T_{\rm rep}$ is inaccessibly low.

Finally, let us compare eq.(3.19) with the corresponding expression on $B_m(0)$. By substituting the parameter values in the dirty limit into the expression[20] $|\mu_0(0)| \propto U_4^{(0)}/(\pi r_B^2 \gamma^{(0)})$ derived in ref.8 (see §2.2 there), we obtain

$$B_m(0) \simeq H_{c2}(0)(1 - d_m(E_{\rm F}\tau)^{-1}), \tag{3.21}$$

where the constant d_m was argued there[8] to be more than 6.0. By taking account of the prospects on $d_{\rm g}^*$-value mentioned above, we believe here that, in general, $B_m(0)$ will lie below $B_{\rm vg}^*$. Further, due to the similarity on parameter dependences in eqs.(3.19) and (3.21), the difference between $B_m(0)$ and $B_{\rm vg}^*$ should enlarge with increasing R_r.

§4. Computation of γ at low T

As mentioned in §3, we have previously[9] shown that $\Gamma(T) \equiv \gamma(T)/\gamma^{(0)}(T = 0)$ at low enough T takes a form of power series[18] in $\lambda_1 \ln(T/T_{cr}^{\rm mf})$ suggestive of the presence of a temperature scale $T_{\rm rep}(\lambda_1)$ of the form (3.18), where $\gamma^{(0)}(T = 0)$ is the limiting value given in eq.(3.17). To demonstrate our argument in §3 on the presence of the T-insensitive intermediate region above $T_{\rm rep}$, we have carried out a numerical calculation of $\Gamma(T)$ useful even at low enough T on the basis of the resummation technique of Oreg and Finkel'stein (OF).[21] Its preliminary results will be briefly reported here.

The OF's technique was originally developed to obtain the reduction of mean field T_c in low dimensional s-wave case due to the interplay between the electron-repulsion and disorder and subsequently, was extended to nonzero field case in ref.19 to examine $H_{c2}(T)$-lines. This method focuses on the selfconsistent equation for the vertex part $\hat{V}_c$ in the Cooper channel and consists

of solving it as a matrix equation in the (Matsubara) frequency space. The mean field transition point is obtained as a vanishing eigenvalue $E_c(\omega = 0)$ of the inverse of $\hat{V}_c(\omega = 0)$, where ω is the Matsubara frequency carried by the pair-field. One can manage to, in order to find Γ valid beyond the lowest order in λ_1, extend this technique to the case with nonzero ω by applying[22] a Pade-approximant to the frequency dependence of E_c.

In Fig4, the Γ v.s. T curves computed for various λ_1 values and at the field $H_{c2}(0)$ of each λ_1 are given. In the temperature range $T/T_{c0} \leq 0.15$ (i.e., $T < T_{cr}^{\rm mf}$) of our interest, the temperature variations of Γ seem to become weaker with increasing λ_1 except at the lowest temperatures. The reduction of γ-values accompanying the λ_1-increase at the intermediate temperatures arises, as well as that of H_{c2}-values, from the high frequency contribution in the diffusion propagators which was neglected for brevity in the γ-expression calculated in ref.9. It is not easy to judge how $T_{\rm rep}$ should be defined from the curves, and it will be defined, for convenience, as the temperature below which Γ becomes less than $\Gamma(T/T_{c0} = 0.15)$. Then, we have, for instance, $T_{\rm rep}(R_r/R_Q = 0.257) \simeq 0.005T_{c0}$, and $T_{\rm rep}(R_r/R_Q = 0.428) \simeq 0.013T_{c0}$. Further, according to Fig.5 in which $R_r/R_Q = 0.428$ is commonly used, the $T_{\rm rep}$ thus defined decreases with decreasing B consistently with eq.(3.18) proportional to $T_{cr}^{\rm mf}$. Based on these figures, we conclude that the presence of an intermediate temperature region, in which γ and hence, $B_{\rm vg}^*$ can be defined as quantities insensitive to T, has been justified by the above microscopic computation.

§5. Summary and Discussion

In this paper, we have examined the position and parameter dependence of 2D VG transition field to be expected in low T limit. In the realistic model with repulsive interaction between electrons, we have a complicated situation due to a *microscopic* T-dependence of the time scale γ becoming remarkable rather *below* a very low temperature scale $T_{\rm rep}$: An intermediate (but low) temperature range above $T_{\rm rep}$, rather than the low T limit below it, is expected to be relevant to experiments suggesting a 2D FSI behavior so that an *apparent* critical field $B_{\rm vg}^*$, being insensitive to T there, plays the role of a "$T = 0$" VG critical field. In a companion paper,[17] we will show how available resistivity data suggesting a $T = 0$ FSI transition can be explained based on the present results and argument.

Finally, we wish to connect the present result with the resistive behavior near the *disorder free* 2D quantum melting line[8] $B_m(T) \simeq B_m(0)$. In ref.8, it was pointed out that, at nonzero temperatures in the quantum regime $T < T_{cr}$, the usual fluctuation conductance results in "fan-shaped" resistivity curves similar to the 2D FSI behavior near or below a fluctuation-corrected $H_{c2}(0)$ and, only close to and below B_m, reduces to the classical vortex flow behavior at the same T. Further, it was argued even that this itself may be the origin of the 2D FSI behavior. This argument based on the neglect of vortex pinnings may be justified only if $B_m > B_{\rm vg}^*$. However, as seen at the end of §3, $B_m(0)$ is in general likely to lie below $B_{\rm vg}^*$. Therefore, to try to understand comprehensively

the FSI behaviors in disordered thin films, a VG fluctuation contribution to conductance needs to be included which will be examined in ref.17. In fact, recent data analysis[1] has suggested a vortex lattice melting to occur much below the critical field B_c at which the resistance becomes flat (see ref.17). We wish to note that, nevertheless, the knowledges[8] on the disorder-free fluctuation conductance at low T become important[17] in understanding differences in the intervening metallic resistance value at $B^*_{\rm vg}$ between various materials.

After submitting this manuscript, we were aware of the paper by Galitski and Larkin[23] who have also examined a quantum transition field on a macroscopic superconductivity in disordered thin films from a different point of view and by neglecting an electron-electron repulsion. Although, in contrast to our approach, they have assumed a spontaneous creation of granular structure, nevertheless an expression determining the transition field at $T = 0$ has been derived which is essentially the same as our eq.(3.16), except for the absence in their expression of eq.(3.16)'s second term (note that $E_{\rm F}\tau$ is denoted as g in ref.23), and has led them to a conclusion similar to our statement given below eq.(3.17) in relation to ref.14. However, the quantum amplitude fluctuation and the microscopic interplay between disorder and an electron-electron repulsion, both of which should contribute to a decrease of a quantum transition field, have been ignored in ref.23. Further, although a glass behavior is expected in lower fields at $T = 0$, the transition field was determined there, according to the sentences below their eq.(16), by assuming an occurrence of the *ordinary* phase coherence in contradiction to our theory with the amplitude fluctuation included showing that the glass ordering is not signaled by a development of the ordinary phase coherence. Explanation of experimental data listed in ref.23 should be ascribed to thermal fluctuation effects.[24,15] Details of this discussion will be given elsewhere.[25]

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Figure Captions

Fig.1

Feynman diagrams contributing to $\mathcal{F}$ and belonging to the same family when classified according to how the diffusion propagators appear. Double solid lines imply the pair-field propagators, a single solid line is an electron propagator, the dotted line with open circle is a diffusion propagator between different replicas, the dotted line with cross denotes a single impurity line carrying $(2\pi N(0)\tau)^{-1}$, and the hatched corner vertex parts imply the Cooperons which modify the couplings to the pair-fields.

Fig.2

Details of the first three diagrams in Fig.1.

Fig.3

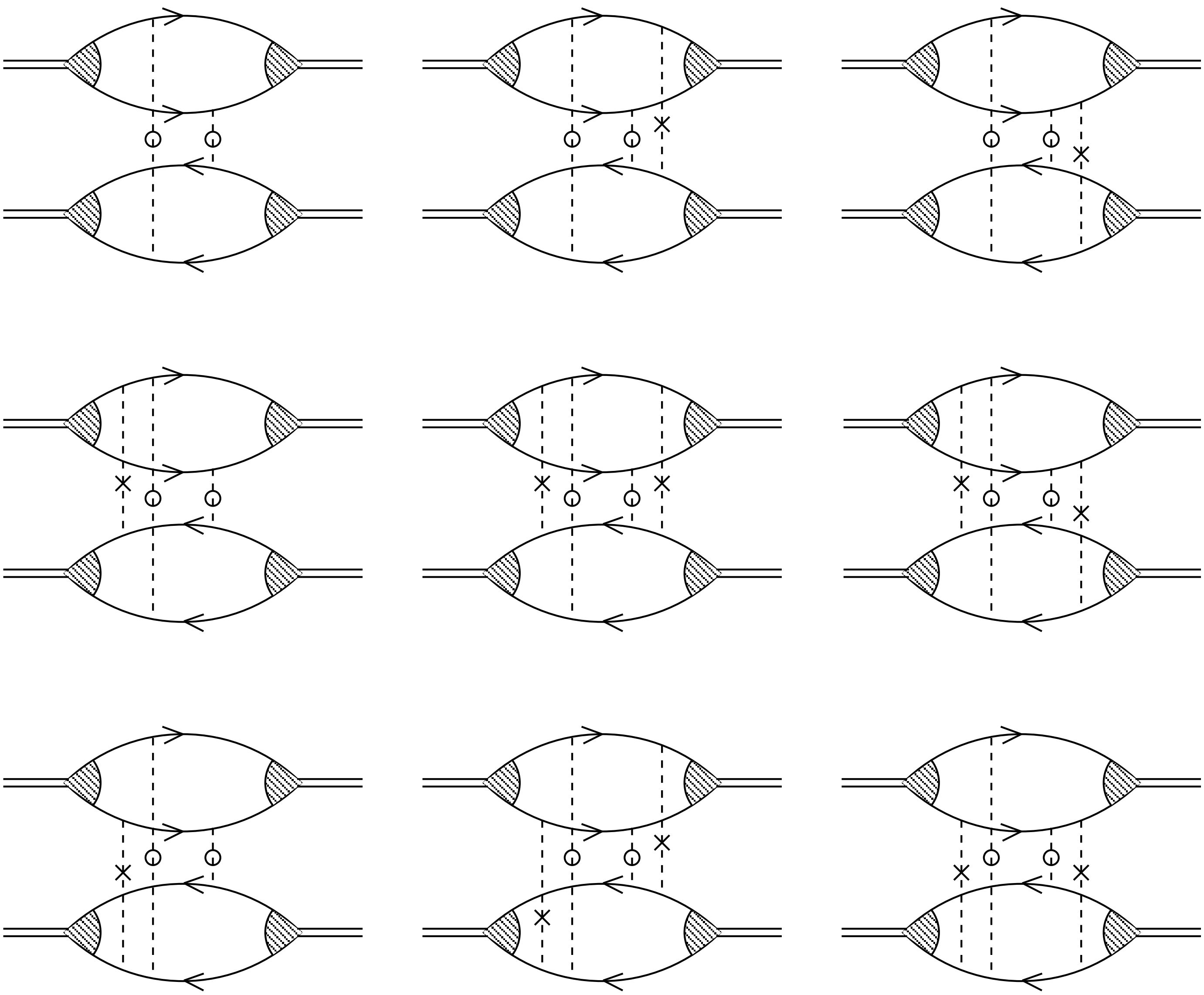
Diagrams necessary in obtaining $B_{\mathrm{vg},0}$, where a hatched corner vertex is defined in (c) (which should not be confused with that in Fig.1 and 2), the double dotted line denotes the pinning line carrying U_p, and the solid circle is the interaction strength carrying U_4. See the text for other details.

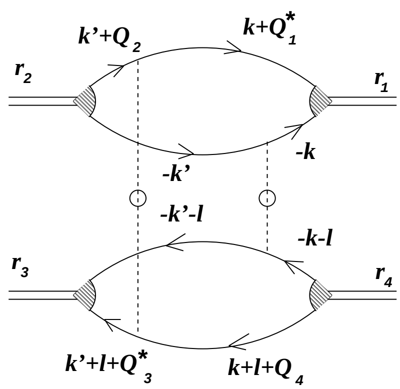
Fig.4

$\Gamma(T)$ v.s. T curves computed in terms of OF's technique for different values of $8\pi\lambda_1 = R_r/R_Q = 0$ (top), 0.086, 0.171, 0.257, 0.342, and 0.428 (bottom). Each curve was obtained by fixing B to the $H_{c2}(0)$-value at each λ_1-value, and $2\pi T_{c0}\tau = 0.25$ was used commonly.

Fig.5

$\Gamma(T)$ v.s. T curves obtained for different field values, $B/H_{c2}(0) = 1.2$ (top), 1.0, and 0.8 (bottom) by fixing $8\pi\lambda_1$ and $2\pi T_{c0}\tau$ to 0.428 and 0.25, respectively.

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k+Q*1
r2
r1
-k'
-k
-k'-l
-k-l
r3
r4
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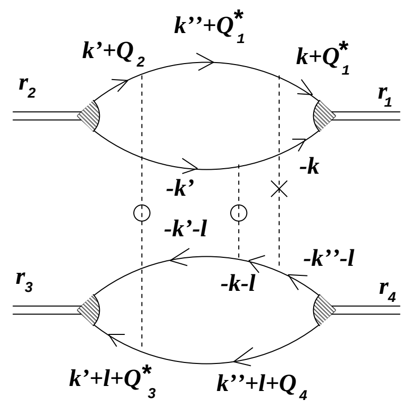
k+l+Q 4

(a)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k'+Q 2
k+Q*1
r2
r1
-k'
-k
-k'-l
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r3
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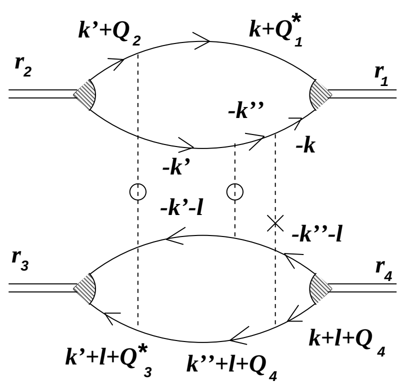
k'+l+Q*3
k''+l+Q 4

(b)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k+Q*1
r2
r1
-k''
-k'
-k
-k'-l
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r3
r4
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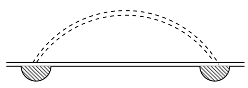
k''+l+Q 4
k+l+Q 4

(c)



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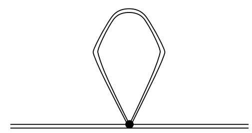
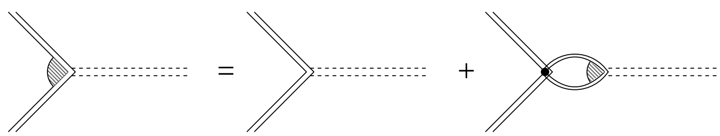
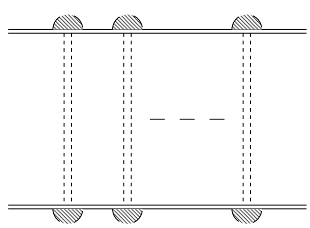


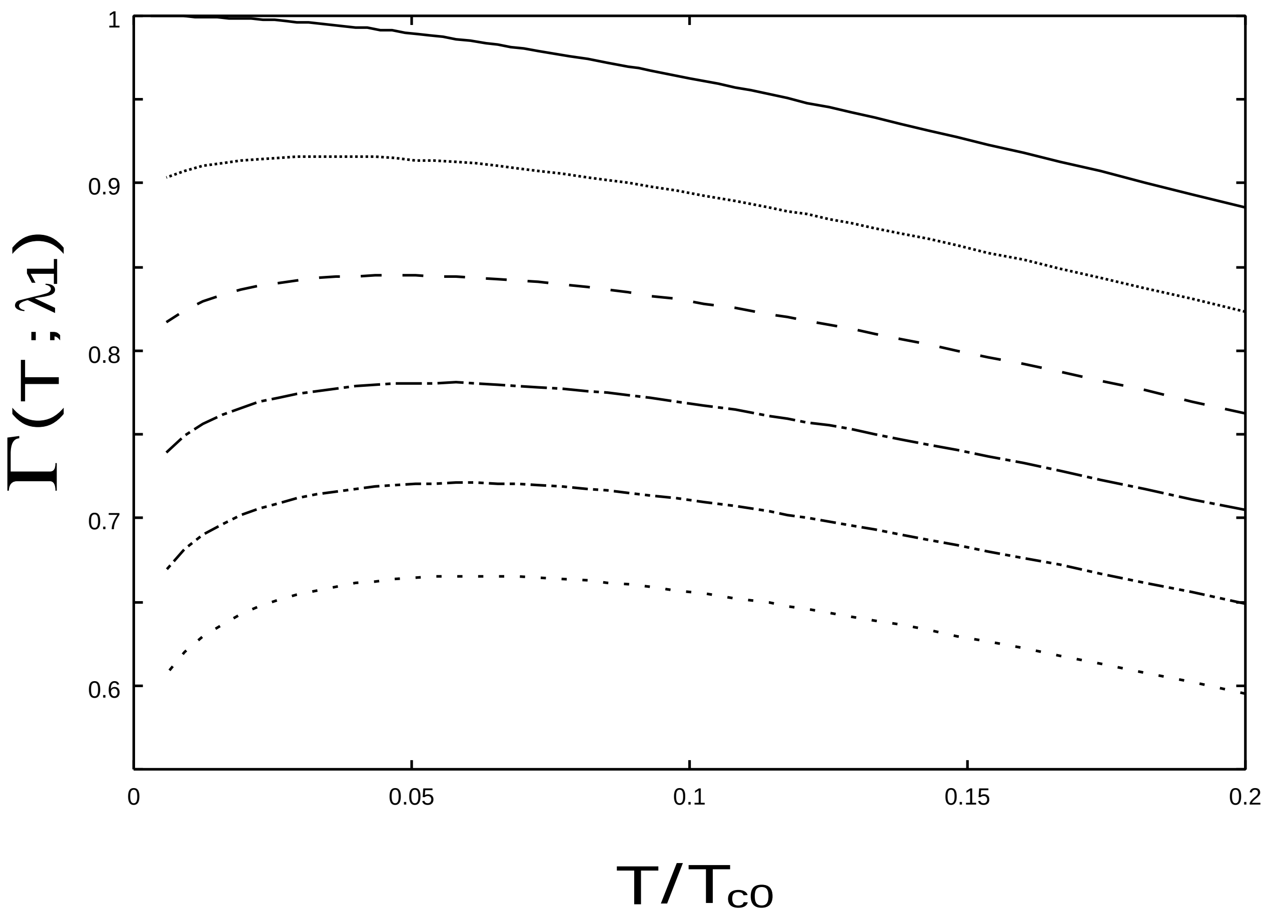
(b)

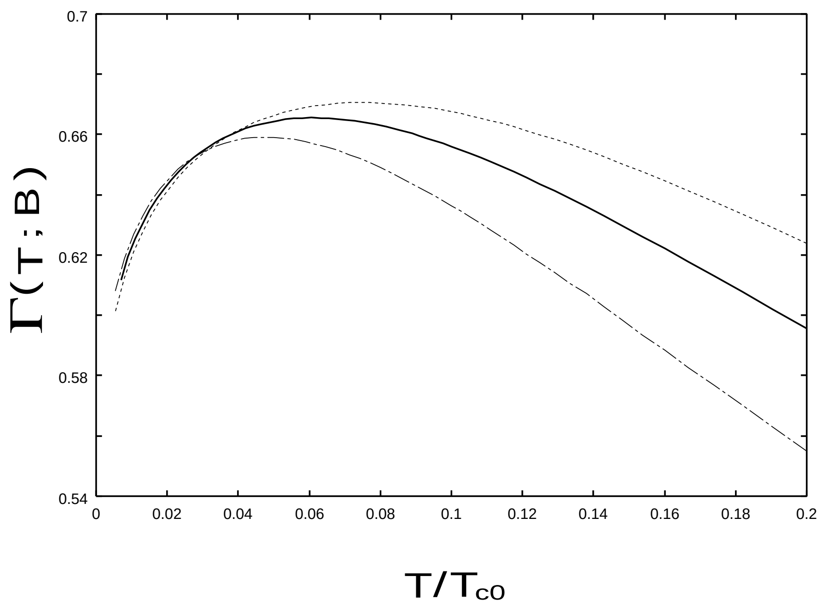


(c)



(d)


1
0.9
0.8
0.7
0.6
0
0.05
0.1
0.15
0.2
$\Gamma(T;\lambda_1)$
T/T_{c0}



0.7
0.66
0.62
0.58
0.54
Γ(T ; B)
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
T/Tc0